Chapter 5 Time Value of Money

The Role of Time Value in Finance (cont.)

- The answer depends on what rate of interest you could earn on any money you receive today.
- For example, if you could deposit the $1,000 today at 12% per year, you would prefer to be paid today.
- Alternatively, if you could only earn 5% on deposited funds, you would be better off if you chose the $1,100 in one year.

Future Value versus Present Value

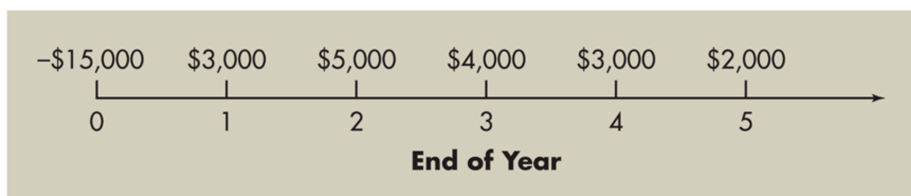
- Suppose a firm has an opportunity to spend $15,000 today on some investment that will produce $17,000 spread out over the next five years as follows:

Year	Cash flow
1	$3,000
2	$5,000
3	$4,000
4	$3,000
5	$2,000

- Is this a wise investment?
- To make the right investment decision, managers need to compare the cash flows at a single point in time.

FIGURE 5.1

Time Line
Time line depicting an investment's cash flows



Simple Interest

With simple interest, you don't earn interest on interest.

Year 1: 5% of $100	=	$5 + $100 = $105
Year 2: 5% of $100	=	$5 + $105 = $110
Year 3: 5% of $100	=	$5 + $110 = $115
Year 4: 5% of $100	=	$5 + $115 = $120
Year 5: 5% of $100	=	$5 + $120 = $125

Compound Interest

With compound interest, a depositor earns interest on interest!

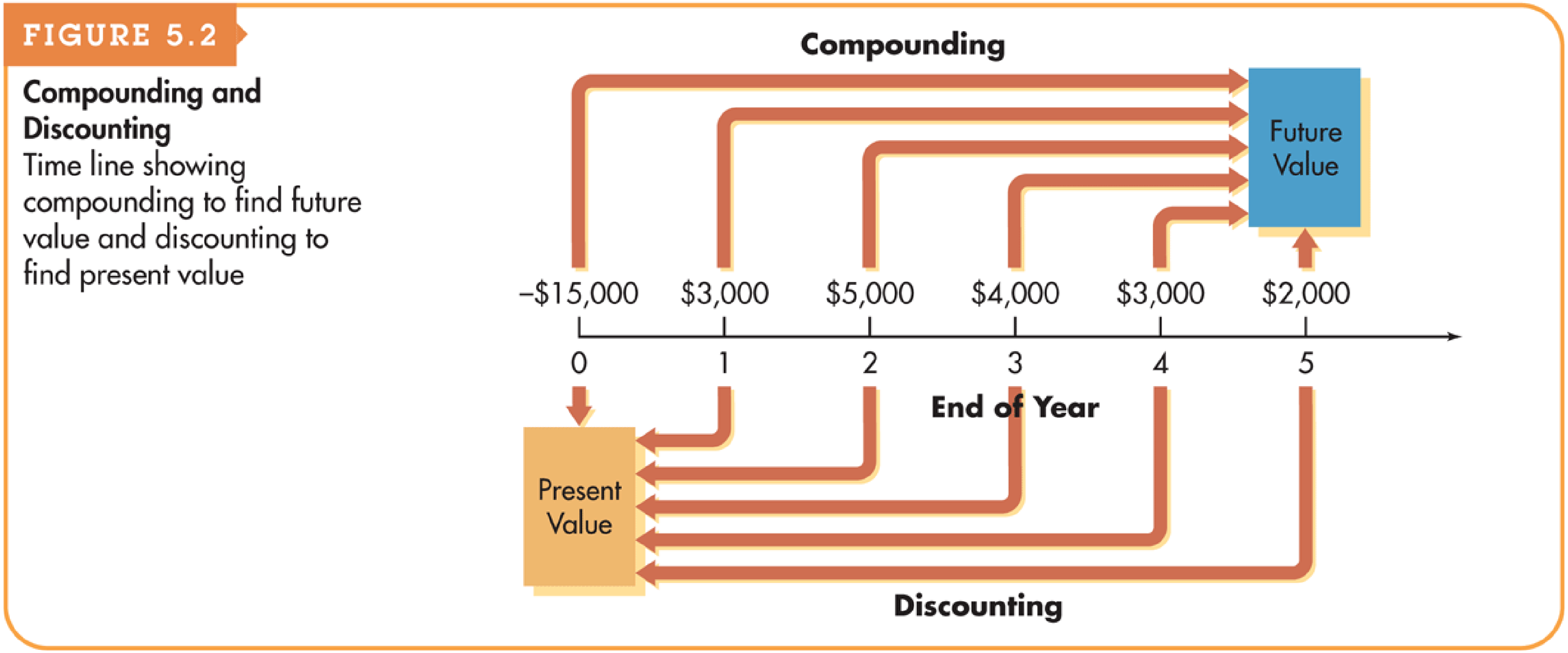
Year 1: 5% of \$100.00	= \$5.00 + \$100.00	= \$105.00
Year 2: 5% of \$105.00	= \$5.25 + \$105.00	= \$110.25
Year 3: 5% of \$110.25	= \$5 .51+ \$110.25	= \$115.76
Year 4: 5% of \$115.76	= \$5.79 + \$115.76	= \$121.55
Year 5: 5% of \$121.55	= \$6.08 + \$121.55	= \$127.63

Khan Academy

http://www.khanacademy.org

https://www.youtube.com/watch?feature=player_embedded&v=-qgdMTbTJlA

Figure 5.2
Compounding and Discounting



Time Value Terms

PV0 = present value or beginning amount

i = interest rate = I/Y

FVn = future value at end of “n” periods

N = years

A = an annuity (series of equal payments or receipts)

- PVA = PV of an annuity
- FVA = FV of an annuity

m = P/Y = periods per year

Future Value of a Single Amount

- **Future value** is the value at a given future date of an amount placed on deposit today and earning interest at a specified rate. Found by applying *compound interest* over a specified period of time.
- **Compound interest** is interest that is earned on a given deposit and has become part of the *principal* at the end of a specified period.
- **Principal** is the amount of money on which interest is paid.

Future values (compound sum)

$$FV = PV * (1 + I/Y)^n = PV * FVIF$$

for now assume annual compounding

PV	I/Y	N	FVIF	FV
1000	10%	4	1.464	1,464.10
250	5	9	1.551	**387.83**
5000	8	20	4.661	**23,304.79**

Note this is an outflow; negative sign on the CF

Future Value (FV)	
Present Value (PV)	**-$1,000.00**
Annual Interest Rate (I/Y)	**10.00%**
Time in Years (N)	**4.00**
Compounding Freq. (m) (P/Y	**1**

Solve for PV	
Solve for FV	**$1,464.10**
Solve for Interest Rate	

Present Value of a Single Amount

- **Present value** is the current dollar value of a future amount—the amount of money that would have to be invested today at a given interest rate over a specified period to equal the future amount.
- It is based on the idea that a dollar today is worth more than a dollar tomorrow.
- **Discounting cash flows** is the process of finding present values; the inverse of compounding interest.
- The discount rate is often also referred to as the opportunity cost, the discount rate, the required return, or the cost of capital.
    - Inflation, growth, interest rate

The Role of Time Value in Finance

- Most financial decisions involve costs & benefits that are spread out over time.
- Time value of money allows comparison of cash flows from different periods.
- Question: Your father has offered to give you some money and asks that you choose one of the following two alternatives:
  - $1,000 today, or
  - $1,100 one year from now.
- What do you do?

Present value (Discounting)

$$PV = FV * (1 / (1 + i)^n) \qquad PV = FV * (PVIF)$$

FV	I/Y	N	PVIF	PV
1000	13%	3	.783	693.05
250	5	8	.677	169.21
5000	8	20	.215	1,072.74

Future Value (FV)	$1,000.00
Present Value (PV)	
Annual Interest Rate (I/Y)	13.00%
Time in Years (N)	3.00
Compounding Freq. (m) (P/Y	1

Solve for PV	-$693.05
Solve for FV	
Solve for Interest Rate	

Your Pension will pay you $100,000 per year when you retire in 25 years. If inflation is 4%, what is that in today's money?

$37,511

Annuities

An **annuity** is a stream of equal periodic cash flows, over a specified time period. These cash flows can be *inflows* of returns earned on investments or *outflows* of funds invested to earn future returns.

- An **ordinary (deferred) annuity** is an annuity for which the cash flow occurs at the *end* of each period
- An **annuity due** is an annuity for which the cash flow occurs at the *beginning* of each period.
- An annuity due will always be greater than an otherwise equivalent ordinary annuity because interest will compound for an additional period.

Table 5.1 Comparison of Ordinary Annuity and Annuity Due Cash Flows ($1,000, 5 Years)

TABLE 5.1 Comparison of Ordinary Annuity and Annuity Due Cash Flows ($1,000, 5 Years)

	Annual cash flows	
Year	Annuity A (*ordinary*)	Annuity B (*annuity due*)
0	$ 0	$1,000
1	1,000	1,000
2	1,000	1,000
3	1,000	1,000
4	1,000	1,000
5	1,000	0
Totals	$5,000	$5,000

Ordinary annuities

FVA = pmt * (FVIFA)

Start at age 20 invest $5000 per year in an IRA until age 60 @ 12%

FVA = 5000(767.080) = 3,835,457

Annual Interest Rate (I/Y)	12.00%
Time in Years (N)	40.00
Compounding Freq. (m) (P/Y	1
Is this an Ordinary Annuity (y	y
Payment (PMT) (A)	-$5,000.00
Growth of an Annuity	
Growth of a Perpetuity	

Solve for FV	$0.00
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	12.00%
PVA	
PMT for PVA	
Interest for PVA	#NUM!
FVA	$3,835,457.10

What if you made monthly payments ($416.67)? $4,901,988

Annuity Due

Staring at age 20, you invest $5,000 per year at the beginning of each year until age 60 @ 12%

FVA= 5,000 * (767.08 * (1+ .12)) = 4,295,648

Annual Interest Rate (I/Y)	12.00%
Time in Years (N)	40.00
Compounding Freq. (m) (P/Y	1
Is this an Ordinary Annuity (y	n
Payment (PMT) (A)	-$5,000.00
Growth of an Annuity	
Growth of a Perpetuity	

Solve for FV	$0.00
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	12.00%
PVA	
PMT for PVA	
Interest for PVA	-100.000%
FVA	$4,295,711.95

Monthly? $4,951,048

Compare to ordinary annuity slide

Retirement example

Need 4,250,000 to retire in 45 yrs can earn 12% interest

How much must you invest monthly for the 45 yrs?

PMT = FVA / FVIFA

Future Value (FV)	$4,250,000.00
Present Value (PV)	
Annual Interest Rate (I/Y)	12.00%
Time in Years (N)	45.00
Compounding Freq. (m) (P/Y)	12
Is this an Ordinary Annuity (y/n)	y
Payment (PMT) (A)	
Growth of an Annuity	
Growth of a Perpetuity	

Solve for PV	-$19,717.28
Solve for FV	
FV (Continuous Compounding)	$0.00
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	12.68%
PVA	
PMT for PVA	$ (198.09)
Interest for PVA (per period)	#NUM!
FVA	
PMT for FVA	$ (198.09)

Retirement example

You have 4,250,000 when you retire. How much can you withdraw monthly as a pension if you expect to live 25 yrs and can earn 10% interest

PVA = PMT * PVIFA

Future Value (FV)	
Present Value (PV)	-$4,250,000.00
Annual Interest Rate (I/Y)	10.00%
Time in Years (N)	25.00
Compounding Freq. (m) (P/Y	12
Is this an Ordinary Annuity (y	y
Payment (PMT) (A)	
Growth of an Annuity	
Growth of a Perpetuity	

Solve for PV	
Solve for FV	$51,242,016.35
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	10.47%
PVA	
PMT for PVA	$38,619.78

What is the value today of $38,619, 45 years from now if inflation is 4%?
$6,402

Matter of Fact

Kansas truck driver, Donald Damon, got the surprise of his life when he learned he held the winning ticket for the Powerball lottery drawing held November 11, 2009. The advertised lottery jackpot was $96.6 million. Damon could have chosen to collect his prize in 30 annual payments of $3,220,000 (30 × $3.22 million = $96.6 million), but instead he elected to accept a lump sum payment of $48,367,329.08, roughly half the stated jackpot total.

What rate of return would Donald need to receive to make the choice between the payments and the lump sum a toss up?

Future Value (FV)	
Present Value (PV)	-$48,367,329.08
Annual Interest Rate (I/Y)	
Time in Years (N)	30.00
Compounding Freq. (m) (P/Y)	1
Is this an Ordinary Annuity (y/n)	y
Payment (PMT) (A)	$3,220,000.00
Growth of an Annuity	
Growth of a Perpetuity	

Solve for PV	
Solve for FV	$48,367,329.08
FV (Continuous Compounding)	$48,367,329.08
Solve for Interest Rate	-100.00%
Solve for Time	
Effective Interest Rate	-100.00%
PVA	$ (96,600,000.00)
PMT for PVA	
Interest for PVA (per period)	5.20%
FVA	
PMT for FVA	
Interest for FVA	5.20%
PV of Perpetuity	#DIV/0!
PV of Growing Annuity	
PV of Growing Perpetuity	#DIV/0!

Growing annuity

You would like to retire with a 100,000 per year income. If you were to live for 35 years in retirement and could earn 8%. How much would you need to acquire in your retirement accounts?

Annual Interest Rate (I/Y)	8.00%
Time in Years (N)	35.00
Compounding Freq. (m) (P/Y	1
Is this an Ordinary Annuity (y	y
Payment (PMT) (A)	$100,000.00
Growth of an Annuity	4.00%
Growth of a Perpetuity	

Solve for FV	$0.00
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	8.00%
PVA	-$1,165,456.82
PMT for PVA	
Interest for PVA	#NUM!
FVA	
PMT for FVA	
Interest for FVA	
PV of Perpetuity	$1,250,000.00
PV of Growing Annuity	-$1,832,770.19

You are concerned about inflation. If you desired for your income to keep pace with a 4% inflation rate, how much would you need?

Present Value of a Perpetuity

A perpetuity is a special kind of annuity.

With a perpetuity, the periodic annuity or cash flow stream continues forever.

PV = Annuity/Interest Rate

For example, how much would I have to deposit today in order to withdraw $1,000 each year forever if I can earn 8% on my deposit?

PV = $1,000/.08 = $12,500

Perpetuities

If you wanted to start an annual endowment that would provide the college and your favorite finance professor with 25,000 per year, and the college could earn 10% per year, how much would you have to donate?

PV = 25,000 / .10 = 250,000

Annual Interest Rate (I/Y)	10.00%
Time in Years (N)	
Compounding Freq. (m) (P/Y	1
Is this an Ordinary Annuity (y	y
Payment (PMT) (A)	$25,000.00
Growth of an Annuity	
Growth of a Perpetuity	4.00%

Solve for FV	$0.00
Solve for Interest Rate	
Solve for Time	#DIV/0!
Effective Interest Rate	10.00%
PVA	$0.00
PMT for PVA	
Interest for PVA	#NUM!
FVA	
PMT for FVA	
Interest for FVA	
PV of Perpetuity	$250,000.00
PV of Growing Annuity	
PV of Growing Perpetuity	-$433,333.33

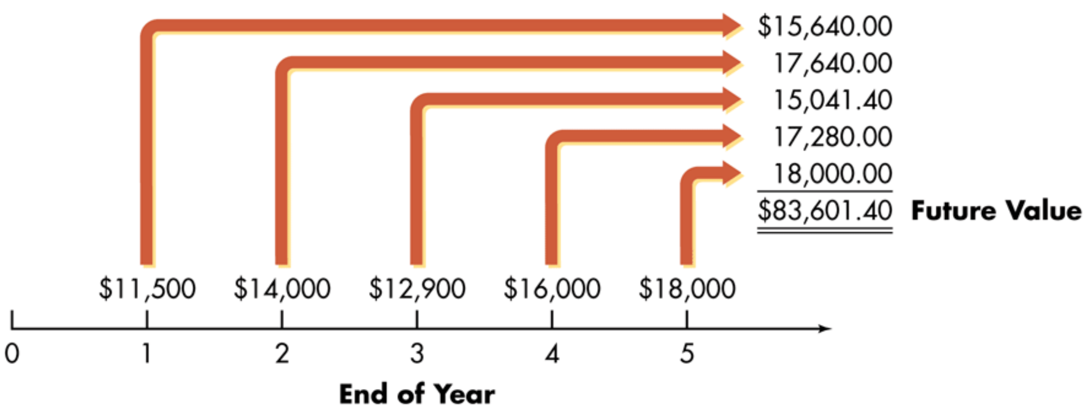
What if you wanted the perpetuity to grow with inflation of 4%?

Mixed Cash Flow Streams

End of year	Cash flow
1	$11,500
2	14,000
3	12,900
4	16,000
5	18,000

Pds	Cash Flow
0	
1	$11,500
2	$14,000
3	$12,900
4	$16,000
5	$18,000
6	
7	
8	
9	
10	
11	

Discount Rate	8.00%
Number of Periods	5
PV of Future Cash Flows	$56,902.30
Net Present Value	$56,902
IRR	#NUM!
FV of Cash Flows	$83,608.15



PV of mixed cash flow

Compounding Interest More Frequently Than Annually

- Compounding more frequently than once a year results in a higher effective interest rate because you are earning on interest on interest more frequently.
- As a result, the effective interest rate is greater than the nominal (annual) interest rate.
- Furthermore, the effective rate of interest will increase the more frequently interest is compounded.

Compounding more frequently

m = P/Y = number of times per year that calculate interest

I/Y = nominal or annual interest (I/Y / m) = periodic rate = table value

N = (years * m) = periods of investment = table value = calculator value

Annual compounding

PV	I/Y	years	FVIF	FV
1000	12%	6	1.973	1973.82

Future Value (FV)	
Present Value (PV)	-$1,000.00
Annual Interest Rate (I/Y)	12.00%
Time in Years (N)	6.00
Compounding Freq. (m) (P/Y)	1
Is this an Ordinary Annuity (y/n)	y
Payment (PMT) (A)	
Growth of an Annuity	
Growth of a Perpetuity	

Solve for PV	
Solve for FV	$1,973.82
FV (Continuous Compounding)	$2,054.43
Solve for Interest Rate	
Solve for Time	
Effective Interest Rate	12.00%
PVA	

What if we compound Interest monthly? M = 12 Why the difference?

Nominal and effective rates

Nominal - stated or contractual int rate, annual interest rate (I/Y)

Effective – EAR -true rate (APR)

$$EAR = \left(1 + \frac{I/Y}{P/Y}\right)^{P/Y} - 1.0$$

i = 12%

m = 1	ieff = 12.00%
m = 2	i = 12.36
m = 4	i = 12.55
m = 12	i = 12.68

Continuous Compounding

- **Continuous compounding** involves the compounding of interest an infinite number of times per year at intervals of microseconds.
- A general equation for continuous compounding

$$FV_n = (PV) \times (e^{i \times n})$$

where *e* is the exponential function.

Special Applications of Time Value: Loan Amortization

- **Loan amortization** is the determination of the equal periodic loan payments necessary to provide a lender with a specified interest return and to repay the loan principal over a specified period.
- The loan amortization process involves finding the future payments, over the term of the loan, whose present value at the loan interest rate equals the amount of initial principal borrowed.
- A **loan amortization schedule** is a schedule of equal payments to repay a loan. It shows the allocation of each loan payment to interest and principal.

Loans

Loan = PVA   100,000

Pmt = PVA / PVIFA   I/Y = 8.75   N = 30

Monthly pmt = 786.70

Loan Amount	**$100,000.00**
Loan Maturity (yrs)	**30**
PMT per Year (P/Y) m	**12**
Annual Interest Rate	**8.75%**

Pmt per Period	**$786.70**
Total AMT Paid	**$283,212.15**
Total Financing Costs	**$183,212.15**

Amortization schedule

Create an amortization schedule

- Scroll right from the loan section

month	PMT	INT	Prin Red	Balance
0				100,000
1	786.70	729.17	57.53	99,942.47
2	786.70	728.75	57.95	99,884.51
3	786.70	728.32	58.38	99,826.14
180	786.70	575.49	211.21	78,713.44

Do you know the rule of 50/25? When you have paid off 50% of the payments of a 30 year mortgage, you have only paid off 25% of the principal!!

Loan Example continued

How long would it take you to pay off this loan if you sent an extra 65.56 per month (1/12 pmt)?

Loan Amount	$100,000.00
Loan Maturity (yrs)	30
PMT per Year (P/Y) m	12
Annual Interest Rate	8.75%
Extra Periodic PMT	$65.56
Biweekly impact =PMT/12	

Pmt per Period	$786.70
Total AMT Paid	$283,212.15
Total Financing Costs	$183,212.15
Impact of Accelerated PMTS	
Years of Loan	22.19
Total AMT Paid	$226,983.70
Interest Saved	$56,228.45

Focus on Practice

New Century Brings Trouble for Subprime Mortgages

- In 2006, some $300 billion worth of adjustable ARMs were reset to higher rates.
- In a market with rising home values, a borrower has the option to refinance their mortgage, using some of the equity created by the home's increasing value to reduce the mortgage payment.
- But after 2006, home prices started a three-year slide, so refinancing was not an option for many subprime borrowers.
- *As a reaction to problems in the subprime area, lenders tightened lending standards. What effect do you think this had on the housing market?*

Solve for interest or return (Growth rates)

Bought an asset 5 yrs ago for $50, now worth $75. What rate of return have you received?

FV = PV (FVIF)   FV/PV=(PVIF)   PVIF= 1.500

I = 8.447

Future Value (FV)	$75.00
Present Value (PV)	-$50.00
Annual Interest Rate (I/Y)	
Time in Years (N)	5.00
Compounding Freq. (m) (P/Y	1

Solve for PV	
Solve for FV	
Solve for Interest Rate	8.45%

Solve for time

How long does it take for an investment to double?

if PV = 1, then FV = 2 I/Y= 8%

FV = PV(FVIF) solve for FVIF = 2

N = 9.006 yrs

$$n = \frac{72}{I}$$

Future Value (FV)	**$2.00**
Present Value (PV)	**-$1.00**
Annual Interest Rate (I/Y)	**8.00%**
Time in Years (N)	
Compounding Freq. (m) (P/Y	**1**

Solve for PV	
Solve for FV	
Solve for Interest Rate	
Solve for Time	**9.01**

You currently earn 50,000 per year and have been able to save $15,000 in a retirement account. You will retire in 35 years at age 60 and inflation is 4%. What will your income need to be in year 1 of retirement to maintain your current lifestyle?

PV = 50K FV = ? N=35 I/Y=4% m=1 PMT = 0

$197,304

If you live to 90, how much do you need in your pension fund at age 60 with 8% return.

$2,221,205

PVA = ? FV = 0 N=30 I/Y=8% m=1 PMT = 197304

If you wanted your retirement income to keep up with an expected inflation rate of 4.5%, how much would you need?

PVA = ? FV = 0 N=30 I/Y=8% m=1 PMT = 197304

Growth of annuity = 4.5%

$3,539,071

How much must you invest each month in your retirement plans to get your desired growing retirement income if you can earn a 12% return?

PV = -15000 FV = 3539071 N=35 I/Y=12% m=12

PMT for FVA = ?

Monthly 397.99

TIME

http://www.youtube.com/watch?feature=player_detailpage&v=_zpGZfFbW4M